For all questions, answer choice (E) NOTA means that none of the given answers is correct. Good Luck!

1. Suppose the position of a particle can be modeled by the function $f(x) = 3x^3 + 18x^2 - 45x + 17$. On what interval is the particle slowing down?
(A) $(-\infty, -5) \cup (-2, 1)$ (B) $(-\infty, -2)$ (C) $(-5, 1)$ (D) $(-\infty, -2) \cup (1, \infty)$ (E) NOTA

2. Which of the following statements is/are always true?
I. If a function is continuous, then it must be differentiable.
II. If the second derivative of a function is equal to zero at point x, then x must be a point of inflection.
III. The integral of a sum of two functions is equal to the sum of their individual integrals.
IV. A right-hand Riemann sum will always be an over approximation for the area under the curve of a monotonically increasing function.

(A) I, II, III, IV (B) II, III, IV (C) III, IV (D) II, III (E) NOTA

3. Evaluate:
$$\int_0^{\ln\sqrt{3}} \frac{e^x}{e^{2x}+1}dx$$
(A) $\frac{\pi}{12}$ (B) $\frac{\pi}{6}$ (C) $\frac{3\pi}{4}$ (D) $\frac{7\pi}{6}$ (E) NOTA

4. Roehl and Shardul are hiking in a forest together when they stumble upon a fork in the road and decide to split up. Roehl walks north in a straight line at a speed of 4 mph and Shardul walks east in a straight line at a speed of 10 mph. What is the rate of change of the distance between Roehl and Shardul 3 hours after they split up?
(A) $2\sqrt{21}$ (B) $6\sqrt{21}$ (C) $2\sqrt{29}$ (D) $6\sqrt{29}$ (E) NOTA

5. Evaluate:
$$\int_{-5}^{3} \frac{x+2}{x^2+15x+54}dx$$
(A) $\frac{4\ln(3)}{3}$ (B) $\frac{7\ln\left(\frac{1}{3}\right)}{3}$ (C) $\frac{4\ln\left(\frac{1}{3}\right)}{3}$ (D) $\frac{\ln\left(\frac{1}{3}\right)}{3}$ (E) NOTA

Use the following table to answer questions $6-8$. Assume that the following functions are differentiable for all reals.

	$x=1$	$x=2$	$x=3$
$f(x)$	4	2	0
$f'(x)$	1	-7	3
$g(x)$	-5	2	1
$g'(x)$	1	2	3

6. Let $h(x)$ be a function defined as $f(g(x))$. What is $h'(3)$?
(A) 1 (B) 3 (C) 9 (D) 12 (E) NOTA

7. Let $i(x)$ be a function defined as $\frac{(f(x))^2}{g(x)}$. What is $i'(2)$?
(A) $-\frac{3}{4}$ (B) $\frac{33}{4}$ (C) 16 (D) -8 (E) NOTA

8. Let $j(x)$ be the inverse function of $f(x)$. What is $j'(4)$?
(A) $\frac{1}{4}$ (B) $\frac{1}{3}$ (C) 1 (D) Cannot be determined (E) NOTA

9. Evaluate:

$$\int_1^e \frac{\ln(x)}{x^5}dx$$

(A) $\frac{e^4-5}{16e^4}$ (B) $\frac{-e^4-5}{16e^4}$ (C) $\frac{e^4+5}{16e^4}$ (D) $\frac{-e^4+5}{16e^4}$ (E) NOTA

10. Consider the function:

$$f(x) = \int_x^{x^2} \frac{e^t}{t^2}dt$$

What is $f'(3)$?
(A) $\frac{2e^9-3e^3}{27}$ (B) $\frac{e^9-9e^3}{81}$ (C) $\frac{e^9-e^3}{9}$ (D) $\frac{2e^9-9e^3}{81}$ (E) NOTA

11. Carson Honéz is a lean mean integrating machine. For his snack, Carson creates a donut by rotating the equation $y = -(x-3)(x-4)$, on $3 \le x \le 4$ about the x-axis, and then about the y-axis. What is the volume of his donut?
(A) $\frac{5\pi}{3}$ (B) $\frac{7\pi}{3}$ (C) $\frac{14\pi}{3}$ (D) $\frac{10\pi}{3}$ (E) NOTA

12. Ms. Pickett wants to build a boat to hold her Cheese-Itz. The boat will be shaped like a rectangular prism with a square base. Since Ms. Pickett only has 100 square feet of paint, she decides to maximize the volume of the boat while still being able to paint all 6 sides of the prism. Assuming the wood has negligible thickness and that all Cheese-Itz are inside of the rectangular prism, what is the maximum volume of Cheese-Itz that the boat can hold in cubic feet?
(A) $\frac{1000\sqrt{2}}{27}$ (B) $\frac{640\sqrt{6}}{9}$ (C) $\frac{95\sqrt{2}}{3}$ (D) $\frac{250\sqrt{6}}{9}$ (E) NOTA

13. Find $\frac{d^2y}{dx^2}$ at $t = 1$ given the parametric equations $x(t) = 3t^2 + 2$ and $y(t) = e^t$.
(A) $\frac{3}{5}$ (B) $-\frac{3}{5}$ (C) $\frac{e}{48}$ (D) $\frac{e}{6}$ (E) NOTA

14. Consider the polar curve $r = 4\sin(\theta) + 2$. What is the slope of the line tangent to the polar curve when $\theta = \frac{\pi}{6}$?
(A) $\frac{3\sqrt{3}}{2}$ (B) $\sqrt{3}$ (C) $\frac{\sqrt{3}}{3}$ (D) $3\sqrt{3}$ (E) NOTA

15. It's midnight and Jasmine is watching shows on Netflix. The rate that Jasmine watches a show is directly proportional to the number of shows she has watched. Suppose that at midnight Jasmine has watched 5 shows, and that at 2 A.M., she has watched 15 shows. Assuming that Jasmine watches shows all night, how many shows has Jasmine watched when its 6 A.M.?
(A) 45 (B) 105 (C) 135 (D) 225 (E) NOTA

16. Given that $f(x) = x^{2017} + \cos(2x)$, what is $f^{2017}\left(\frac{\pi}{4}\right)$?
(A) $2017! - 2^{2017}$ (B) $\frac{(2017!)\pi}{4} - 2^{2017}$ (C) $2017!$ (D) $2017! + 2^{2017}$ (E) NOTA

17. Approximate $\sqrt{50}$ using Eulers method with two steps of equal width and a starting point of $\sqrt{49} = 7$.
(A) $\frac{197}{28} + \frac{25\sqrt{198}}{99} - \frac{\sqrt{198}}{4}$ (B) $\frac{187}{7} + \frac{25\sqrt{198}}{198} - \frac{\sqrt{198}}{4}$
(C) $\frac{197}{7} + \frac{50\sqrt{198}}{99} - \frac{\sqrt{198}}{8}$ (D) $\frac{187}{28} + \frac{25\sqrt{198}}{99} - \frac{\sqrt{198}}{8}$ (E) NOTA

18. Find $\frac{dy}{dx}$ at $(1, 0)$ given $5 + 2x^2y + 3x\sin(y) = 3y^3x + 5x$.
(A) 0 (B) $-\frac{3}{5}$ (C) $\frac{2}{5}$ (D) $-\frac{17}{5}$ (E) NOTA

19. What is the constant term of $f''(x)$ given $f(x) = \left(\frac{1}{x} + x^2\right)^4$?

(A) 0 (B) 12 (C) -36 (D) -108 (E) NOTA

20. Give the interval of convergence for the following series:

$$\sum_{n=1}^{\infty} \frac{6(x+9)^n}{4^{2n} \cdot n}$$

(A) $[-25, 7]$ (B) $[-25, 7)$ (C) $(-13, -5]$ (D) $(-13, -5)$ (E) NOTA

21. What is the sum of the first three non-zero terms of the Maclaurin series expansion for $\arccos(x)$?

(A) $\frac{\pi}{2} + x - \frac{x^3}{2}$ (B) $\frac{\pi}{2} + x - \frac{x^3}{6}$ (C) $\frac{\pi}{2} - x - \frac{x^3}{6}$ (D) $\frac{\pi}{2} - x - \frac{x^3}{2}$ (E) NOTA

22. Which of the following series converge absolutely?

I. $\sum_{n=1}^{\infty} (-1)^n * 2017^{\frac{1}{n}}$

II. $\sum_{n=2}^{\infty} \frac{(\ln(n))^2}{n}$

III. $\sum_{n=1}^{\infty} \frac{n^{2017}}{e^n}$

IV. $\sum_{n=1}^{\infty} \frac{1}{\pi^n + e}$

(A) I, II, III (B) II, III, IV (C) III, IV (D) I, III, IV (E) NOTA

23. What is the maximum volume of a right circular cone that can be inscribed in a sphere with a radius of 2?

(A) $\frac{128\pi}{27}$ (B) $\frac{128\pi}{81}$ (C) $\frac{256\pi}{27}$ (D) $\frac{256\pi}{81}$ (E) NOTA

24. Evaluate: $\int_1^2 \frac{2x^2 + 3}{x^4 + 2x^3 + x} dx$

(A) $\frac{7}{12}$ (B) $\ln(\frac{7}{12})$ (C) $8\ln 2 - \ln 17$ (D) $5\ln 2 - \ln 17$ (E) NOTA

25. Sohan and Qing are trying to decide who gets to ask Arden Cho™ to homecoming. They both randomly generate a number from (0, 1) and if Sohan's number is greater than the square of the inverse sine of Qing's number, then Sohan wins and gets to ask Arden Cho™ to homecoming. What is the probability that Sohan wins the previous number game?

(A) $\sin^{-1}(\frac{\sqrt{2}}{5})$ (B) $\sin^{-1}(1) + \cos^{-1}(1)$ (C) $\sin(1) + \cos(1)$ (D) $1 - 2\sin(1) + 2\cos(1)$ (E) NOTA

26. Given the polynomial of minimum degree that can be determined by the points:

- $F(-1) = -4$
- $F(0) = 0$
- $F(1) = 0$
- $F(2) = 8$
- $F(3) = 60$

Find the value of the 4th derivative of $F(x)$ at $x = 3$.

(A) 0 (B) 12 (C) 24 (D) 6 (E) NOTA

27. Given,

$$E(a,b) = R(b) - \int_0^a f(t)dt$$

where $R(b)$ represents the right-hand Riemman sum of $f(x)$ from $(0,a)$ into b sub-intervals. If $f(x) = x$, find the smallest value of b, $b > 1$ such that,

$$\lfloor E(10,b) \rfloor < 2$$

(A) 26 (B) 51 (C) 2017 (D) Does Not Exist (E) NOTA

28. The history of Calculus is often overshadowed by the actual mathematics behind Calculus, despite being just as important. Identify this 17th century French mathematician that claimed Calculus "was a collection of ingenious fallacies," but later recanted. Ironically, his namesake proof explains one of the most basic concepts of Calculus, and is the starting point for many other proofs.
(A) Isaac Newton (B) Gottfried Leibniz (C) Michel Rolle (D) Leonhard Euler (E) NOTA

Use the following information to answer Questions 29-30

As expressions become increasingly complicated, it becomes impractical to use l'Hôpital's rule repeatedly to evaluate limits. Fortunately, adopting the Big-O notation, a method of approximating functions using other functions, can yield promising results. The notation, written as $O(f(x))$, can be interpreted as such:

$$\text{As } x \to \alpha, \text{ if } f(x) = O(g(x)), \text{ then the } \lim_{x\to\alpha} \frac{f(x)}{g(x)} \le M$$

In other words, the polynomial $\dfrac{f(x)}{g(x)}$ is bounded by M. For example, as $x \to \infty$, $x = O(2x)$, and $x^2 = O(x^3)$.

29. Big-O notation can also be used to approximate complicated functions from its Maclaurin series. For example,

$$\text{As } x \to 0, \ \frac{1}{1-x} = 1 + x + x^2 + O(x^3)$$

which is valid since $\displaystyle\lim_{x\to 0} \frac{x^3 + x^4 + \dots}{x^3} = 1$ and is therefore bounded. Using this idea, evaluate the following limit:

$$\lim_{x\to 0} \frac{(x + 2x^3 + 2x^5) - xe^{2x^2}}{x^7}$$

(A) 0 (B) $-\dfrac{4}{3}$ (C) $\dfrac{1}{8}$ (D) Does Not Exist (E) NOTA

30. Now, evaluate:

$$\lim_{x\to 0} \frac{x \sin x^3}{1 - \cos(\frac{3}{2}x^2)}$$

(A) -1 (B) $\dfrac{2}{3}$ (C) $\dfrac{8}{9}$ (D) Does Not Exist (E) NOTA